SOLUTION OF EXERCISE # 5.4

Exercise # 5.4

In any triangle ABC, by using the law of Cosines:

Q.1: $a = 56,\ c = 30,\ \beta = 35^\circ$, find b.

Sol. By using law of cosines:

$$b^2 = c^2 + a^2 - 2ca\cos\beta$$
$$b^2 = (30)^2 + (56)^2 - 2(30)(56)\cos 35^\circ$$
$$b^2 = 900 + 3136 - 2752.35$$
$$b^2 = 1283.65 \Rightarrow \sqrt{b^2} = \sqrt{1283.65} \Rightarrow \boxed{b = 35.83}$$

Q.2: $b = 25, \quad c = 37, \quad \alpha = 65^\circ$ find a.

Sol. By using law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$a^2 = (25)^2 + (37)^2 - 2(25)(37)\cos 65^\circ$$
$$a^2 = 625 + 1369 - 781.84$$
$$a^2 = 1212.16$$
$$\sqrt{a^2} = \sqrt{1212.16} \Rightarrow \boxed{a = 34.82}$$

Q.3: $b = 5, \quad c = 8, \quad \alpha = 60^\circ$ find a. **(IIA-2017)**

Sol. By using law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$a^2 = (5)^2 + (8)^2 - 2(5)(8)\cos 60^\circ$$
$$a^2 = 25 + 64 - 40$$
$$a^2 = 49$$
$$\sqrt{a^2} = \sqrt{49} \Rightarrow \boxed{a = 7}$$

Q.4: $a = 212, \quad c = 135, \quad \beta = 37^\circ 51'$ find b.

Sol. By using law of cosines:

$$b^2 = c^2 + a^2 - 2ca\cos\beta$$
$$b^2 = (135)^2 + (212)^2 - 2(135)(212)\cos 37^\circ 51'$$
$$b^2 = 18225 + 44944 - 45197.84$$
$$b^2 = 17971.16$$
$$\sqrt{b^2} = \sqrt{17971.16} \Rightarrow \boxed{b = 134.06}$$

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Q.5: $a = 16$, $b = 17$, $\gamma = 25^\circ$ **find c.**

Sol. By using law of cosines: $c^2 = a^2 + b^2 - 2\,ab\cos\gamma$

$c^2 = (16)^2 + (17)^2 - 2(16)(17)\cos 25^\circ$

$c^2 = 256 + 289 - 493.03$

$c^2 = 51.97$

$\sqrt{c^2} = \sqrt{51.97} \Rightarrow \boxed{c = 7.21}$

Q.6: $a = 44$, $b = 55$, $\gamma = 114^\circ$ **find c.** (IIA-2021)

Sol. By using law of cosines: $c^2 = a^2 + b^2 - 2ab\cos\gamma$

$c^2 = (44)^2 + (55)^2 - 2(44)(55)\cos 114^\circ$

$c^2 = 1936 + 3025 - (-11968.61)$

$c^2 = 6929.61$

$\sqrt{c^2} = \sqrt{6929.6} \Rightarrow \boxed{c = 83.24}$

Q.7: $a = 13$, $b = 10$, $c = 17$, **find** α **&** β.

Sol. By using law of cosines:

$a^2 = b^2 + c^2 - 2bc\cos\alpha$

$2bc\cos\alpha = b^2 + c^2 - a^2$

$$\cos\alpha = \frac{b^2+c^2-a^2}{2bc} = \frac{(10)^2+(17)^2-(13)^2}{2(10)(17)} = \frac{100+289-169}{340}$$

$\cos\alpha = 0.6471$

$\alpha = \cos^{-1}(0.6471) \Rightarrow \boxed{\alpha = 49^\circ 40'47''}$

Now again by using law of cosines again.

$b^2 = c^2 + a^2 - 2ca\cos\beta$

$2ca\cos\beta = c^2 + a^2 - b^2$

$$\cos\beta = \frac{c^2+a^2-b^2}{2ca} = \frac{(17)^2+(13)^2-(10)^2}{2(17)(13)}$$

$$\cos\beta = \frac{289+169-100}{442} = 0.8099$$

$\beta = \cos^{-1}(0.8099) \Rightarrow \boxed{\beta = 35^\circ 54'30''}$

Q.8: **Three villages P, Q and R are connected by straight roads. Measure PQ is 6km and the measure QR is**

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9km. The measure of the angle between PQ and QR is 120º. Find the distance between P and R.

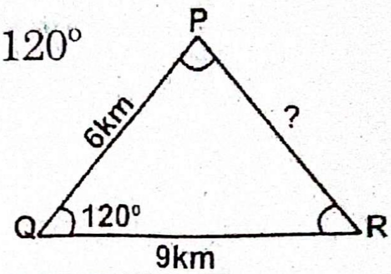
Sol. By using law of cosines:

$$(\overline{PR})^2 = (\overline{PQ})^2 + (\overline{QR})^2 - 2(\overline{PQ})(\overline{QR})\cos PQR$$

$$(\overline{PR})^2 = (6)^2 + (9)^2 - 2(6)(9)\cos 120^\circ$$

$$(\overline{PR})^2 = 36 + 81 - (-54)$$

$$(\overline{PR})^2 = 171$$



$$\sqrt{(\overline{PR})^2} = \sqrt{171} \Rightarrow \boxed{\overline{PR} = 13.80\text{km}}$$

Q.9: **Two points A & B are at distance 55 & 32m respectively from a point P. The measure of angle between AP and BP is 37º. Find the distance between B and A.**

Sol. From the figure.

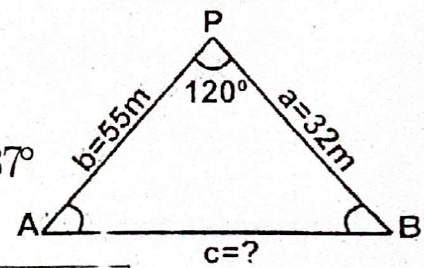
$b = 55\text{m},\ a = 32,\ \gamma = 37^\circ$ & Let $\overline{AB} = c$

By using law of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

$$c^2 = (32)^2 + (55)^2 - 2(32)(55)\cos 37^\circ$$

$$c^2 = 1024 + 3025 - 2811.20$$



$$c^2 = 1237.80 \Rightarrow \boxed{c = |\overline{AB}| = 35.18\text{m}}$$

Q.10: **Find the cosine of the smallest measure of an angle of a triangle with 12, 13 and 14m as the measure of its sides.** **(IA-2021), (IIA-2021), (IA-2022)**

Sol. Let $a = 12,\quad b = 13,\quad c = 14$

As side 'a' is smallest, so we will find α ?

$$\cos\alpha = \frac{b^2 + c^2 - a^2}{2bc} = \frac{(13)^2 + (14)^2 - (12)^2}{2(13)(14)} = \frac{221}{364} = 0.6071$$

$$\alpha = \cos^{-1}(0.6071) \Rightarrow \boxed{\alpha = 52^\circ 37'}$$